



Fractional Brownian dynamics in naira/dollar foreign exchange rates

Fractional
Brownian
dynamics

191

Olusegun Felix Ayadi

*Department of Accounting and Finance, Texas Southern University, Houston,
Texas, USA*

Abstract

Purpose – This paper seeks to characterize the behavior of the naira/dollar foreign exchange rate series over the period 1999 through 2006 to determine if the process generating the series has long memory which is a special case of fractional Brownian motion. The existence of long memory contradicts the notion of market efficiency.

Design/methodology/approach – The paper employs the modified rescaled range R/S test which is proposed by Lo to test the null hypothesis that daily and weekly NGN/USD exchange rates from 1999 through 2006 exhibit short-memory process. The second test that was also employed is the Geweke-Porter-Hubak (GPH) test which was refined by Hurvich *et al.*

Findings – The results show that long memory is present in daily and weekly foreign exchange level series of the Nigerian naira for the period sampled. This evidence implies that the Nigerian foreign exchange market may not be efficient. Thus, it is possible for investors to realize abnormal profit by taking an investment position based on predicted exchange rates. The results reported in this paper are also indicative of a deviation from long-run PPP.

Originality/value – This paper is the first to empirically apply the modified R/S and GPH tests to explore the existence of long-memory process in a country study of foreign exchange series using data from Nigeria.

Keywords Long-memory, Foreign exchange, Fractional Brownian motion, Rescaled R/S test, Nigeria
Paper type Research paper

1. Introduction

A given time series is said to be governed by some underlying process. A logical question is whether or not the times series possesses some patterns or regularity which can be examined through econometric modeling. A fractional Brownian motion is a stochastic process which possesses stationary increments, self-similarity as well as long-range dependence (Nualart, 2002). A fractional Brownian motion is also associated with regularity of sample paths. The study of long memory in time series data has been in existence for quite some time. A renewed application in the financial economics field had recently gained ground as noted in Peters (1994, 1996), Mandelbrot (1997), Granger and Joyeux (1980), Beran (1993) and Bouchaud *et al.* (1999). As far back as 1900 Bachelier argues that financial asset prices can be described as a random walk because fluctuations in these prices are assumed to follow a Gaussian probability distribution (Bouchaud *et al.*, 1999). The focus of researchers was a development of reliable model for predicting fluctuations which is crucial for managing the risks inherent in asset prices.

The literature on long memory addresses the correlation of times series at long lags (Box-Steffensmeier and Tomlinson, 1999). Mandelbrot (1972) raises the issue of long memory in stock prices. The thesis of this area of study is based on the premise that actual movements in time series are stochastically influenced by the recent to the most remote past. Furthermore, with an autoregressive moving average (ARMA) model, one can make more accurate forecasts of foreign exchange rates when long memory exists.



According to Huang and Yang (1999), the foundation of capital market equilibrium lies on the efficient market hypothesis. Under this “theory,” financial time series are independently and identically log-normally distributed. Thus, an investigation of efficiency in the foreign exchange markets is based on whether or not exchange rate changes are serially correlated (Haubrich, 1993; Hsieh, 1988).

Olisadebe (1991, 1995) notes, that the real exchange rate in Nigeria, like in any economy, is affected by economic fundamentals, which have both domestic and international aspects. He notes that the strength of any economy is reflected on the strength of its currency. Given the rate at which the Nigerian naira has been depreciated over the years, the author reports that a segment of the Nigerian population has reached a conclusion that the Nigerian currency is “bastardized” and “traumatized.” An anecdotal implication of the remark by Olisadebe is that the Nigerian foreign exchange market is efficient. In view of the foregoing, this paper seeks to characterize the behavior of the naira/dollar foreign exchange rate series over the period, 1999 through 2006 to determine if the process generating the series has long memory. The existence of long memory contradicts the notion of market efficiency.

2. Long memory in financial time series

Summers (1986) provides, one of the strongest challenges to the tests of efficient market hypothesis by challenging the early tests for their low power. According to him, serial correlation of time series at short horizon could not reveal all mean-reverting characteristics of asset prices. This argument is supported by Fama and French (1988) whose model stock prices as a sum of a random walk and a stationary component.

A time series variable is generally classified stationary or non-stationary. Stationarity means that a time series will return to a constant mean in which case random shocks to the series are forgotten at a consistent rate (Lebo, 2001). A series is non-stationary if it wanders around without regularly returning to a particular value such as the mean. In this case, the best estimate of a distant value depends critically on the current value of series. In general, non-stationary variables do not have mean values because their long-run average values do not converge. A good example of a non-stationary process is a random walk model. Lebo (2001) notes, that a stationary series contains only short-term memory.

The decision to model a time series either as stationary or non-stationary has important consequences. A spurious result arises if a non-stationary series is modeled as a stationary series. Employing fractional integration methods can mitigate this problem. Crato and de Lima (1994), observe that several economists agree that financial time series have a long memory because actual movements in the series are stochastically influenced by the recent to the most remote past. Long memory in a time series is indicative of persistence in the volatility of the series.

Haubrich and Lo (2001), pose the question: are time series better approximated by fluctuations around a deterministic trend or by a random walk plus a stationary component? Mandelbrot (1971) first suggests the possibility that asset prices could exhibit long-range dependence or long memory in time series. Granger and Ding (1996), define a long-memory series based on a slowly declining autocorrelation structure. According to Cheung and Lai (2001), long-memory dynamics can confound unit root tests and undermine their ability to distinguish between low- and high-frequency dynamics. They note that long-memory dynamics are properly taken into consideration when a researcher employs fractional time series methodology. Green (2000) notes that the autocorrelation of a stationary process decays at an exponential

rate, thus, large values typically cease to appear after a few lags. On the other hand, the autocorrelation of a non-stationary series remain persistently very high at long lags.

3. Monetary model of exchange rate determination

The monetary model of foreign exchange determination is based of some simplifying assumptions, which include the presence of perfect capital mobility, perfect substitutability of foreign and domestic bonds, interest rate parity conditions and the existence of purchasing power parity (PPP). There are three versions of the monetary model: flexible price monetary model, sticky price model and sticky price model augmented with relative price differential. In this paper, the focus is on a version suggested by Hou (1999). According to Hou, the exchange rate is a price and thus, it is subject to the forces of demand and supply. Its determination is based on the following equation:

$$E_0 = \frac{FC}{DC} \quad (1)$$

where E_0 is the prevailing exchange spot exchange rate; FC is foreign currency; DC is domestic currency

$$FC = \Delta A_d + X \quad (2)$$

$$DC = \Delta A_f + M \quad (3)$$

where ΔA_d is the inflow of assets denominated in domestic currency into foreign countries; M is imports denominated in domestic currency; ΔA_f = outflow of assets denominated in foreign currency; X is foreign price value of exports.

Equation (1) can now be rewritten as:

$$E_0 = \frac{\Delta A_d + X}{\Delta A_f + M} \quad (4)$$

According to Hou, two features of the exchange rate determination are obvious from Equation (4). The horizon of exchange rate determination and the flow of assets are brought into fore. The fundamentals are responsible for determining the long-run exchange rate as shown with the presence of imports and exports in the equation. In the short-run, the impact of financial variables may dominate the determination of exchange rates. In general, financial time series such as stock prices and exchange rate contain two components: the non-random persistent component and the transitory component (Fama and French, 1988; Poterba and Summers, 1988). That is:

$$P_t = P_t^* + u_t \quad (5)$$

where

$$P_t^* = P_{t-1}^* + \varepsilon_t \quad (6)$$

P_t^* is the permanent component; u_t is transitory component.

The authors argue that the permanent component follows a random walk while the transitory component of the series is persistent in the sense that positive values tend to be followed by further positive values. More importantly, present results which

suggest that a good model for exchange rate series should incorporate serial correlation, time-varying variance, long-memory, peakedness and fat tails.

There is a body of knowledge that seeks to test the concept of PPP of exchange rates. According to Ahking (2010), recent attempts to test PPP have focussed on the long run in view of frequent and large and persistent deviations from PPP. The issue at stake is whether deviations from PPP are transitory or permanent. Most of the empirical tests take the form of tests for stationarity of the real exchange rate. If time series of the real exchange rate is stationary, then deviations from PPP are said to be transitory. On the other hand, a unit-root non-stationarity in time series of the real exchange rate would imply that deviations from PPP are permanent. PPP is a financial concept which refers to the equivalency in the purchasing power of two currencies in their respective domestic economies once exchange rates are taken into consideration.

4. The Nigerian foreign exchange market

The Central Bank Act of 1959, the Exchange Control Act of 1962 and Decrees 24 and 25 of 1991 charge the Central Bank of Nigeria (CBN) with the maintenance of a healthy balance of payments position and a stable exchange rate. The CBN documents the objectives of exchange rate policy as to preserve the value of the domestic currency, maintain a buoyant external reserve position, ensure internal and external balance and achieve macroeconomic stability.

Prior to the establishment of the CBN, the private sector earned foreign currencies, which were deposited with foreign commercial banks. These foreign commercial banks acted as agents of local exporters because there was no foreign exchange market at the time. Moreover, convertibility of the Nigerian pound was not an issue because the Nigerian pound was tied at par to the British pound sterling. Nnanna (2002) notes, that this mechanism provided an effective way of maintaining Nigeria's balance of payments and also controlling inflation. The parity between the Nigerian and British currencies was discontinued in 1967 when the British pound was devalued.

With the termination of parity between the Nigerian currency and the pound sterling, Nigeria decided to peg its currency to the US dollar. The international financial crisis which started in early 1970 and the subsequent devaluation of the US dollar led Nigeria to abandon the dollar peg. Nigeria went back to the British pound until 1973 when the US dollar became the currency of choice for a second time. It should be noted that the Nigerian currency underwent a *de facto* devaluation in sympathy with the US dollar in 1973 and 1975. This was the genesis for the desire on the part of the Nigerian monetary authorities to manage the exchange rate. In 1978, the naira was pegged to a basket of currencies, representing Nigeria's major trading partners. According to Nnanna (2002), there was a considerable stability in the naira exchange rate until 1981 when Nigeria was faced with a serious economic crisis.

Prior to 1986, Ahmed (1990) observes that the Nigerian currency was substantially overvalued. An overvalued currency favors imports of goods and services. It discourages exports and thus weakens the current account balance of a country. Nigeria's output as well as export of agricultural products fetched a limited amount of hard currencies, thus, leading to a regime of import licensing due to a fall in inflows of foreign exchange. Domestically produced goods were less competitive relative to imports (Rasheed, 1995).

In 1986, the Nigerian government pursued a structural adjustment program (SAP) in the face of a persistent current account deficits, rising external indebtedness, overvalued currency, distortions in relative prices leading to a loss of international

competitiveness, a recession in economic growth, rising inflation and inefficient allocation of resources (Omoruyi, 1990). Among other policy measures, the government deregulated the foreign exchange market. Since 1986, the Nigerian monetary authorities had tried different foreign exchange management strategies. Between 1986 and 1987, a dual exchange rate regime was instituted. There was a government-controlled exchange rate for official transactions and the second rate was more or less market driven. In January 1989, the official and market rates were merged under the inter-bank foreign exchange market (IFEM). The IFEM tried various pricing mechanisms ranging from “marginal rate” pricing system to the “average of successful bids” in a Dutch auction approach.

In March 1992, the exchange rate pricing mechanism was revamped again just immediately after the monetary authorities devaluated the naira. According to Usman (2001), the rationale for the slight devaluation was to close the gap between the rates in the parallel market and the main foreign exchange market. Shortly after the 1992 devaluation, there was a renewed demand pressure and speculative activities that again, widened the gap. In addition to this, the economy performed miserably in 1994. By January 1995, the autonomous foreign exchange market (AFEM) was introduced to replace IFEM. The AFEM was a system of heavily managed float (Abdullahi, 2012). In October 1999, the AFEM was replaced by a daily IFEM one more time. IFEM was a flexible but was managed by the monetary authorities. In July 2002, the daily IFEM was replaced with a Dutch auction system (DAS). According to a local publication of the First City Monument Bank, the fundamental problems of foreign exchange scarcity and rate misalignment cannot be resolved by the current DAS. The CBN was blamed for holding the exchange rate fixed for a long time under IFEM when market forces dictated otherwise. IFEM was a daily market and would have been a better alternative to DAS, which, opened only twice weekly. In 2006, the CBN replaced DAS with a two-way quote system referred to as wholesale Dutch auction system (WDAS). Under WDAS, authorized dealers are allowed to trade on their own accounts. Authorized dealers are also given the opportunity to trade in the inter-bank market. The introduction of WDAS also coincides with banking consolidation in Nigeria. According to Olowe (2009), Arene and Odusolu (1998), Ezike and Amah (2011) and Mojekwu *et al.* (2011), the period from financial liberalization of 1986 through 2006 witnessed extreme managed floating exchange rate regimes which resulted in exchange rate uncertainty. Therefore, it is logical to investigate the underlying generating process in the foreign exchange market from the emergence of Nigerian civilian administration in 1999 through the introduction of WDAS in 2006.

5. Empirical analysis

Time series processes can be classified by their dependence nature into short-range dependence (short-memory) or long-range dependence (long-memory). The identification of each classification can be determined by observing the behavior of autocorrelation function $\rho(k)$, of the process at different lags, k . Stationary processes possess short-memory if the autocorrelation function exhibits an exponential decay. On the other hand, a stationary stochastic process has long-memory if the autocorrelation function decays slowly at a hyperbolic rate. Thus, if $\rho(k) \sim L(k)k^{2d-1}$ as $k \rightarrow \infty$ then the process possesses long memory. In this simple analysis, d is the degree of integration, it is also referred to as the memory parameter. For example, $d = 0$ for a stationary process while and $I(d = 1)$ process is a process that can be made stationary after it is differenced once. An $I(d = 1)$ process possesses a unit root. In relation to the popular Hurst exponent, H ,

the expression, $d=2-H$ captures the relationship between d and H . In the case of $0.5 < d < 1$, the process is said to be non-stationary with a non-invertible ARMA representation. The process can also exhibit mean reversion as its autocorrelation function decays to zero. If $d > 1$ holds, then the process becomes non-mean-reversion. For a process with $0 < d < 0.5$ there is long-memory as the autocorrelation function does not possess a finite sum. The attempt in this section is to determine the value of either d or H .

5.1 Data

This research paper relies on historical time series data spanning the period 1999 through 2006. The daily and weekly exchange rates are defined as the nominal effective trade-weighted exchange rate index with a 1985 base of 100. The series are collected from various publications of the CBN and the International Financial Statistics published by the International Monetary Fund. The sample period is chosen to start with the 1999 advent of democratic administration through 2006 when the Nigerian banking industry was consolidated. The sample period is described by many scholars as a highly managed floating exchange rate period. Two econometric methods that are applied to the data are modified rescaled range analysis and Geweke and Porter-Hudak (GPH, 1983) test.

5.2 Modified rescaled range (R/S) analysis

This approach is a modification of the classical R/S model by Hurst (1951). The focal point is to test for long-term memory effects vs short-term memory alternatives such as ARCH effects and other non-normal processes (Hays *et al.*, 2000). This paper employs the modified R/S technique suggested by Lo (1991) to explore the presence of long-term dependence in monthly exchange rate data on the Nigerian currency from 1986 through 2004. Using Bhar's (1996) variables, the continuously compounded return, x_t between time $t-1$ and t for the spot currency prices, P_t and P_{t-1} is defined as:

$$x_t = \ln P_t - \ln P_{t-1} \tag{7}$$

The return generating process is given by:

$$x_t = \mu + \varepsilon_t \tag{8}$$

where μ is the time invariant constant term; ε_t is an error term that exhibits short-term serial dependence.

Using both daily and weekly level as well as return series, the following modified rescaled R/S statistic is calculated as in Lo (1991):

$$Q_n = \left[\max \sum_{j=1}^k (x_j - \bar{x}_n) - \min \sum_{j=1}^k (x_j - \bar{x}_n) \right] / \sigma_n(q) \tag{9}$$

Lo (1991) standardized Q_n by dividing its calculated value by \sqrt{n} and denoted this standardized value as $V_n(q)$ with q (bandwidth) defined as the integer part of $[4(T/100)^{0.25}]$. The standardized value is compared with critical values of 2.098, 1.862 and 1.747 representing 1, 5 and 10 percent levels of significance, respectively. It should be noted that the null hypothesis is that the series exhibits short-term memory. If this

null hypothesis is rejected, then the series is said to exhibit long memory or long-term dependence.

5.3 The GPH method

GPH (1983) developed a semi-parametric method to obtain an estimate of the fractional differencing parameter of long-memory series. The GPH estimate of long-memory parameter, d , is the least squares coefficient obtained in a regression of the log periodogram on $X_j - \log(2 - 2 \cos(\omega_j)) \approx -\log \omega_j^2$ for $j=1, 2, \dots, J$ with $\omega_j = 2\pi j/T$ and $J < T$. T is the sample size. GPH employed $J = T^{0.5}$ to test for the presence of statistically significant long-range dependence using a simple Student t -test of the estimate of d . The null hypothesis of no long-memory is defined as $d=0$. Hurvich *et al.* (1998) conclude that the power of the GPH test is greatly improved when J is set equal to $T^{0.8}$.

6. Empirical results

The times series employed in this study are displayed graphically in Figures 1 through 4. The descriptive statistics for the sampled series are summarized in Table I. Both the daily and weekly level series show negative skewness while the daily and weekly series are positively skewed. All the series exhibit non-normality as shown in the values of the Jarque-Bera test.

Table II gives the results of the modified R/S analysis of daily and weekly naira exchange rates from January 1999 through December 2006. The exchange rate is defined as the nominal effective trade-weighted exchange rate index with a 1985 base of 100. The test statistic, $Vn(q)$ is significant for daily and weekly level series but not for the return series. Thus, the null hypothesis of short-term memory is rejected in favor of long-term memory for daily and weekly level series. Therefore, one can

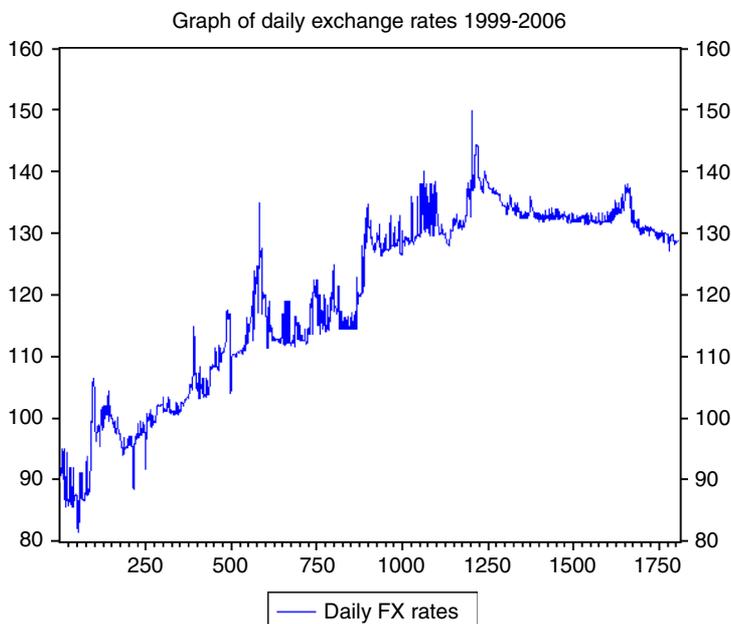


Figure 1.
Daily NGN-USD exchange
rate returns

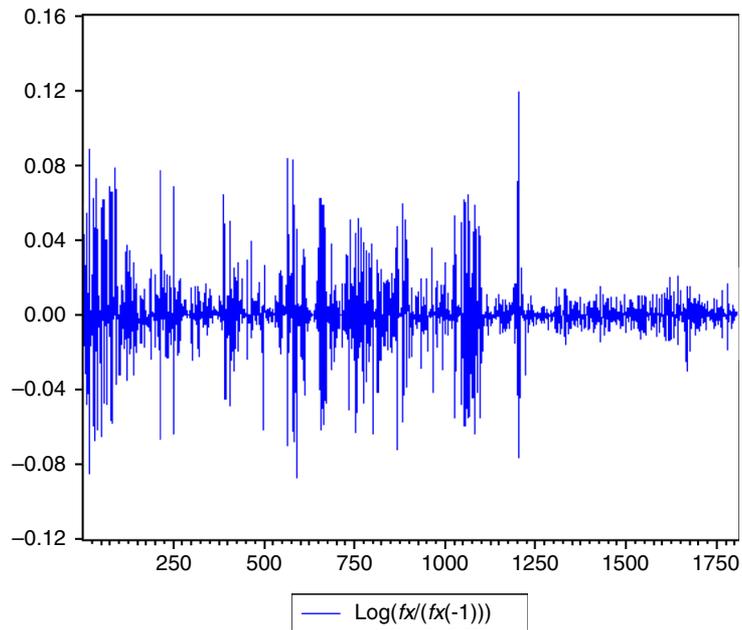


Figure 2.
Daily FX continuous
return series 1999-2006

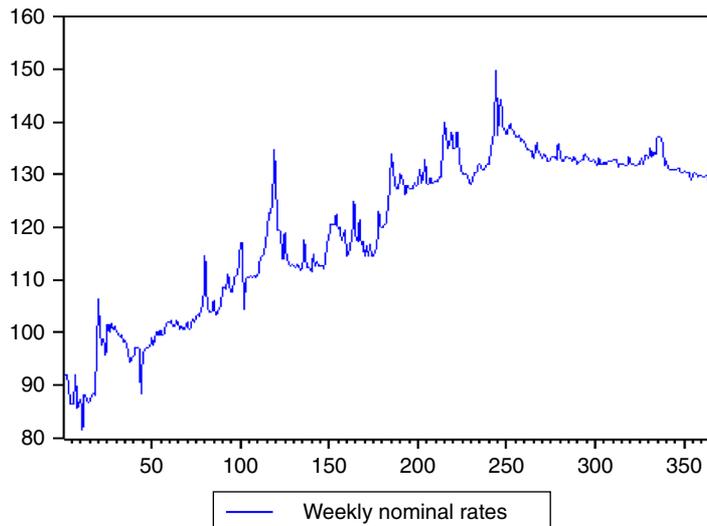


Figure 3.
Weekly nominal exchange
rates 1999-2006

conclude the level of foreign exchange rates in Nigeria for the period 1999 through 2006 exhibits long memory. However, the return series do not possess long memory.

Table III shows the results of the GPH test. The point estimates of d are positive for daily and weekly level series and negative for return series. The results for the level series are statistically significant at the 1 percent level but only the weekly return series' results are statistically significant at the 5 percent level. These results provide

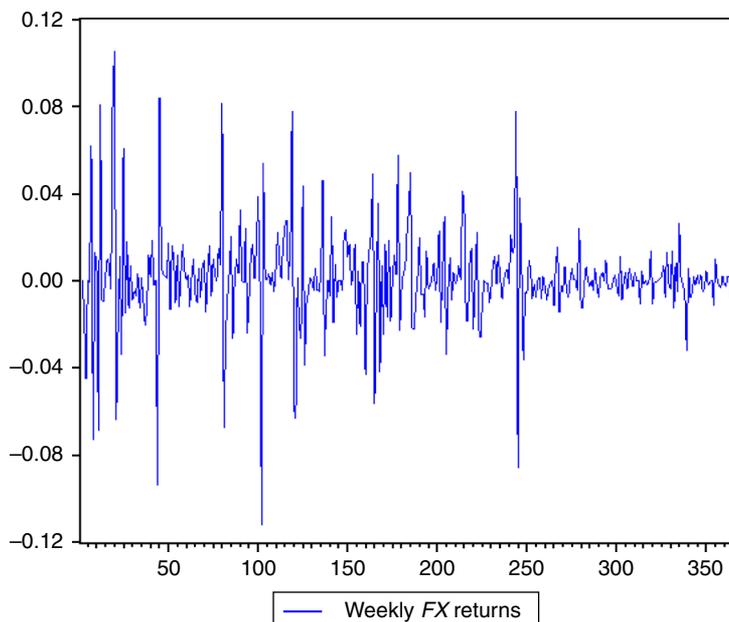


Figure 4.
Weekly exchange rate
returns 1999-2006

Statistic	Daily level	Weekly level	Daily return	Weekly return
Mean	119.7798	119.7694	0.000193	0.000920
Median	127.2500	127.2000	0.000000	0.000000
Maximum	149.9250	149.6500	0.119832	0.105460
Minimum	81.5000	81.5000	-0.087510	-0.112132
SD	14.7914	14.9705	0.016274	0.022859
Skewness	-0.5952	-0.5865	0.227325	0.046592
Kurtosis	2.1538	2.1512	12.26217	9.225691
Jarque-Bera	160.7036 (0.0000)	31.9725 (0.0000)	6,474.674 (0.00000)	589.5953 (0.00000)
Observations	1,808	366	1,807	365

Notes: Exchange rates are reported as NGN-USD. The Jarque-Bera statistic tests for normality with the number in bracket representing the probability associated with the null hypothesis of normality. If the associated probability is “small” then the null hypothesis of normality is rejected

Table I.
NGN-USD exchange
rate summary statistics
(1999-2006)

Time series	$V_n(q)$	Bandwidth (q)
Daily exchange rates levels	6.1905*	8
Weekly exchange rate levels	3.4068*	5
Daily exchange rate returns	1.0411	8
Weekly exchange rate returns	1.3182	5

Note: *Indicates statistical significance at the 1 percent conventional levels

Table II.
Modified rescaled range
[$V_n(q)$] statistic For
NGN-USD exchange rates

evidence for positive long memory in naira foreign exchange rate series. Only the weekly exchange rate series show evidence of long memory.

7. Summary and conclusion

The subject of long memory in foreign exchange rates has implications for volatility. On the other hand, exchange rate volatility has adverse effect on international trade and capital flows. Exchange rate volatility leads to an increase in risks associated with importing and exporting. It can also have a destabilizing effect on the economy by impairing economic welfare.

This paper attempts to investigate the presence of long memory in foreign exchange series covering the period 1999 through 2006. The statistical tool employed in this investigation is the modified rescaled range analysis developed by Lo (1991) and the adjusted GPH (1983) methodology. The rescaled range methodology and the adjusted GPH test are robust to the presence of heteroscedasticity in time series.

The results show that long memory is present in daily and weekly foreign exchange level series of the Nigerian naira for the period sampled. This evidence implies that the Nigerian foreign exchange market may not be efficient. Thus, it is possible for investors to realize abnormal profit by taking an investment position based on predicted exchange rates. The results reported in this paper also are indicative of a deviation from long-run PPP.

Usman (2001) reports the existence of a substantial arbitrage premium between the official rates and parallel market rates. The results presented in this paper, is consistent with the conclusion reached by Usman. It should be noted that the Nigerian foreign exchange market was under a significant pressure. The monetary authorities adopted a demand management strategy because the mono-product nature of the Nigerian economy. In other words, the main source of supply of foreign exchange to this market is oil whose supply is controlled by OPEC.

According to Usman (2001), the demand pressure and the subsequent depreciation of the naira under IFEM was due to the expansionary fiscal policy of the government. This is also responsible for the persistent excess liquidity in the banking system. The presence of excess liquidity is responsible for the upward movement in exchange rate against the naira. As this takes place, the manufacturing enterprises, which depend so much on imported materials are forced to incur increased production costs, which is the major source of inflation in Nigeria. In addition to the aforementioned situation the foreign exchange market has become a victim of capital flight, speculative activities of market players and non-transparency of some authorized dealers. A major step needed to correct the ills in the Nigerian foreign exchange market is for the federal government to grant complete independence to the CBN.

Series	<i>d</i>	GPH statistic
Daily exchange rate levels	1.0238	8.9628*
Weekly exchange rate levels	0.8276	14.9259*
Daily exchange rate returns	-0.1407	-1.2322
Weekly exchange rate returns	-0.2447	-4.4121**

Table III.
GPH results

Notes: *,**Significance at the 1 and 5 percent levels, respectively

References

- Abdullahi, S.A. (2012), "Impact of foreign exchange intervention of exchange rate, external reserve and economic in Nigeria", Proceedings of the International Conference on Excellence in Business, University of Sharjah, Sharjah, May 9-10.
- Ahking, F.W. (2010), "Non-parametric tests of real exchange rates in the post-Bretton Woods era", *Empirical Economics*, Vol. 39 No. 2, pp. 439-456.
- Ahmed, A. (1990), "Perspectives on medium to long-term outlook for the Nigerian economy", *CBN Economic and Financial Review*, Vol. 28 No. 4, pp. 37-47.
- Arene, C.J. and Odusolu, G.O. (1998), "Exchange rate reform as a policy instrument for stimulating small-holder agricultural investment demand and output supply in Nigeria", *Economic Affairs*, Vol. 43 No. 4, pp. 232-240.
- Beran, J. (1993), "Fitting long-memory models by generalized linear regression", *Biometrika*, Vol. 80 No. 4, pp. 817-822.
- Bhar, R. (1996), "Testing for long-term memory in yen/dollar exchange rate", working paper, University of Western Sydney, Milperra.
- Bouchaud, J.P., Cizeau, P., Laloux, L. and Potters, M. (1999), "Mutual attractions: physics and finance", *Physics World*, Vol. 12 No. 1, pp. 25-29.
- Box-Steffensmeier, J.M. and Tomlinson, A.R. (1999), "Fractional integration methods in political science", working paper, The Ohio State University, Columbus, OH.
- Cheung, Y.W. and Lai, K.S. (2001), "Long memory and nonlinear mean reversion in Japanese yen-based real exchange rates", *Journal of International Money and Finance*, Vol. 20 No. 1, pp. 115-132.
- Crato, N. and de Lima, P.J.F. (1994), "Long-range dependence in the conditional variance of stock returns", *Economics Letters*, Vol. 45 No. 3, pp. 281-285.
- Ezike, J.E. and Amah, P.N. (2011), "Time series variation of foreign exchange in the Nigerian wholesale Dutch auction market", *Journal of Economics and International Finance*, Vol. 3 No. 13, pp. 685-696.
- Fama, E.F. and French, K.R. (1988), "Permanent and temporary component of stock prices", *Journal of Political Economy*, Vol. 96 No. 2, pp. 246-273.
- Geweke, J. and Porter-Hudak, S. (1983), "The estimation and application of long memory time series models", *Journal of Time Series Analysis*, Vol. 4 No. 4, pp. 221-238.
- Granger, C.W.J. and Ding, Z. (1996), "Varieties of long memory models", *Journal of Econometrics*, Vol. 73 No. 1, pp. 61-78.
- Granger, C.W.J. and Joyeux, R. (1980), "An introduction to long-memory time series models and fractional differencing", *Journal of Time Series Analysis*, Vol. 1 No. 1, pp. 15-29.
- Green, W.H. (2000), *Econometrics Analysis*, 4th ed., Prentice Hall, Upper Saddle River, NJ, pp. 785-788.
- Haubrich, J.G. (1993), "Consumption and fractional differencing: old and new anomalies", *Review of Economics and Statistics*, Vol. 75 No. 4, pp. 767-772.
- Haubrich, J.G. and Lo, A.W. (2001), "The sources and nature of long-term memory in aggregate output", *FRB of Cleveland, Economic Review*, Vol. 37 No. 2, pp. 15-30.
- Hays, P., Schreiber, M., Payne, J.E., Ewing, B.T. and Piette, M.J. (2000), "Are net discount ratios stationary? Evidence of mean-reversion and persistence", *The Journal of Risk and Insurance*, Vol. 67 No. 3, pp. 439-449.
- Hou, Y. (1999), "Exchange rate determination: an empirical test", working paper, Yale University, New Haven, CT.
- Hsieh, D.A. (1988), "The statistical properties of daily foreign exchange rates: 1974-1983", *Journal of International Economics*, Vol. 24 Nos 1-2, pp. 129-145.

- Huang, B.N. and Yang, C.Y. (1999), "The impact of financial liberalization on stock price volatility in emerging markets", *Journal of Comparative Economics*, Vol. 28 No. 2, pp. 321-339.
- Hurst, H.E. (1951), "Long-term storage capacity of reservoirs", *Transactions of the American Society of Civil Engineers*, Vol. 116 No. 3, pp. 770-799.
- Hurvich, C.M., Deo, R. and Brodsky, J. (1998), "The mean squared error of Geweke and Porter-Hubak's estimator of the memory parameter of a long-memory time series", *Journal of Time Series Analysis*, Vol. 19 No. 1, pp. 19-46.
- Lebo, M.J. (2001), "Divided government, united approval: long memory and the dynamics of congressional and presidential approval", working paper, M.I.T. Data Center, Harvard, MA.
- Lo, A.W. (1991), "Long-term memory in stock market prices", *Econometrica*, Vol. 59 No. 5, pp. 1279-1313.
- Mandelbrot, B.B. (1971), "When can price be arbitrated efficiently? A limit to the validity of the random walk and Martingale models", *Review of Economics and Statistics*, Vol. 53 No. 3, August, pp. 225-236.
- Mandelbrot, B.B. (1972), "Statistical methodology for non-periodic cycles: from the covariance to R/S analysis", *Annals of Economic and Social Measurement*, Vol. 1 No. 3, pp. 259-290.
- Mandelbrot, B.B. (1997), *Fractals and Scaling in Finance*, Springer, New York, NY.
- Mojekwu, J.N., Okpala, O.P. and Adeleke, I.A. (2011), "Comparative analysis of models of monetary exchange rates in Nigeria", *African Journal of Business Management*, Vol. 5 No. 34, pp. 13025-13029.
- Nnanna, O.J. (2002), "Monetary policy and exchange rate stability in Nigeria", paper presented at Nigerian Economic Society Seminar, Lagos, May 23.
- Nualart, D. (2002), "Fractional brownian motion", available at: www.probabilidad.org.mx/Actividades/vii-symposium/Abstracts/David_Nualart/on (accessed December 10, 2004).
- Olisadebe, E.U. (1991), "Appraisal of recent exchange rate policy measures in Nigeria", *CBN Economic and Financial Review*, Vol. 29 No. 2, pp. 156-185.
- Olisadebe, E.U. (1995), "The role of the central bank of Nigeria: current issues in Nigeria's exchange rate policy", *CBN Economic and Financial Review*, Vol. 33 No. 4, pp. 317-332.
- Olowe, R.A. (2009), "Modeling naira/dollar exchange rate volatility: application of GARCH and asymmetric models", *International Review of Business Research Papers*, Vol. 5 No. 3, pp. 377-398.
- Omoruyi, S.E. (1990), "Exchange rate policy and the structural adjustment programme in African countries", *CBN Economic and Financial Review*, Vol. 28 No. 4, pp. 48-54.
- Peters, E. (1994), *Fractal Market Analysis*, John Wiley and Sons, New York, NY.
- Peters, E. (1996), *Chaos and Order in the Capital Markets*, John Wiley and Sons, New York, NY.
- Poterba, J.M. and Summers, L.H. (1988), "Mean reversion in stock prices: evidence and implications", *Journal of Financial Economics*, Vol. 22 No. 1, pp. 27-59.
- Rasheed, M.R. (1995), "Foreign exchange management in Nigeria", *CBN Economic and Financial Review*, Vol. 33 No. 4, pp. 352-356.
- Summers, L.H. (1986), "Does the stock market rationally reflect fundamental values?", *Journal of Finance*, Vol. 41 No. 3, pp. 591-601.
- Usman, S. (2001), "Interest and exchange rate management in Nigeria", paper presented at the Banking Seminar organized by the Institute of Directors, Nigeria (IOD), Lagos, August 7.

Further Reading

Cheung, Y.W. and Lai, K.S. (1993), "A fractional cointegration analysis of purchasing power parity", *Journal of Business and Economic Statistics*, Vol. 11 No. 1, pp. 103-112.

Omoruyi, S.E. (1987), "A review of the structural adjustment programme, the foreign exchange market and trade policies in Nigeria", *CBN Economic and Financial Review*, Vol. 25 No. 4, pp. 29-38.

About the author

Olusegun Felix Ayadi is JP Morgan Chase Professor of Financial Literacy in the Jesse H. Jones School of Business at Texas Southern University, Houston, Texas, USA. He served as Interim Associate Dean at the Jesse H. Jones School of Business from September 2010 through November 2011. He is editor of *Southwestern Business Administration Journal (SBAJ)*. He was a Fulbright Scholar at Covenant University in Nigeria from January through August 2012. His research interests are in the areas of investments and international finance, especially issues relating to emerging markets. Olusegun Felix Ayadi can be contacted at: ayadi_fo@tsu.edu